Dynamic Keynesian Models of Monetary and Fiscal Stabilization Policies

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Abstract

In this paper, we summarize a series of dynamic Keynesian models of monetary and fiscal stabilization policies that were presented in our previous works. All models are formulated by systems of nonlinear differential equations. We take up three models sequentially. The first model is the simplest two-dimensional model of monetary stabilization policy without debt effects. The second model is more complicated four-dimensional model of monetary stabilization policy with private debt effect. The third model is the most general six-dimensional model of monetary and fiscal stabilization policy mix with twin (private and public) debt effects. These models produce various types of dynamic behaviors (stable, unstable, and cyclical behaviors) according to the crucial parameter values.

Keywords: Dynamic Keynesian Model, Monetary and Fiscal Stabilization Policies, Nonlinear Dynamics, Hopf Bifurcation.
JEL Classification: E12, E31, E32, E52.

1 Introduction

It is well known among the researchers of financial economics that Minsky's(1975, 1982,1986) 'financial instability hypothesis', which means that a financially dominated capitalist economy is inherently unstable, became more and more influential in recent times. The credibility of this hypothesis is growing rapidly because of the recent economic turbulence in the world economy including the US, the European and the Japanese experiences.[1]

Minsky did not develop his own mathematical models of financial instability, but some mathematical-oriented economists developed mathematical models that formalize Minsky's idea.[2] It is important to note, incidentally, that Minsky(1986) pointed out that the central bank and the 'big government' can stabilize the inherent unstable economy by adopting the appropriate monetary and fiscal policy mix. In fact, we also developed a series of dynamic Keynesian models of monetary and fiscal stabilization policies that are based on Minsky's basic idea.[3]

[1]Krugman(2012) and Wolf(2015) provide some useful descriptions of the US and the European experiences. As for the interpretation of the Japanese experience, see Asada (ed.)(2014)(in particular, the papers by A. Noguchi, M. Wakatabe and T. Asada in this volume).

[2]See, for example, Charpe, Chiarella, Flaschel and Semmler(2011), Chiarella, Flaschel and Franke(2005), Gong(2005), Nasica(2000), and Semmler(ed.)(1989).

[3]See, for example, Asada(2010, 2012, 2014), Asada and Ouchi(2015), Asada, Demetrian and Zimka(2016, 2018, 2019), and Murakami and Asada(2018).

In this paper, we summarize the essences of our three papers on Keynesian macroeconomic stabilization policy that are based on the idea of Misky's financial instability hypothesis. All models are formulated by means of systems of nonlinear differential equations. We take up three models sequentially. In section 2, we summarize the simplest two-dimensional model of monetary stabilization policy without debt effect that was developed by Asada, Demetrian and Zimka(2016). In section 3, we take up more complicated four-dimensional model of monetary stabilization policy with private debt effect that was developed by Asada, Demetrian and Zimka(2018). Section 4 summarizes the most general six-dimensional model of monetary and fiscal stabilization policy mix with twin (private and public) debt effects by Asada, Dmetrian and Zimka(2019). These models produce various types of dynamic behaviors (stable, unstable, and cyclical behaviors) according to the crucial parameter values. In the final section, we restate Asada, Demetrian and Zimka's(2019) economic interpretation of the analytical results of the most general six-dimensional model of monetary and fiscal stabilization policy mix.

2 A Two-dimensional Model of Monetary Stabilization Policy without Debt Effects

In this section, we take up the simplest prototype model of monetary stabilization policy without debt effects, which was presented by Asada(2010) for the first time and later investigated by Asada, Demetrian and Zimka(2016) thoroughly mathematically and numerically. This model consists of the following system of static and dynamic equations.

$$Y = Y(r-\pi^e, G, \tau)\,; \quad Y_{r-\pi^e} = \frac{\partial Y}{\partial(r-\pi^e)} < 0,\ Y_G = \frac{\partial Y}{\partial G} > 0,\ Y_\tau = \frac{\partial Y}{\partial \tau} < 0 \tag{1}$$

$$\frac{M}{p} = L(Y, r, \pi^e)\,; \quad L_Y = \frac{\partial L}{\partial Y} > 0,\ L_r = \frac{\partial L}{\partial r} < 0,\ L_{\pi^e} = \frac{\partial L}{\partial \pi^e} < 0 \tag{2}$$

$$\pi = \varepsilon(Y - \overline{Y}) + \pi^e\,; \quad \overline{Y} > 0,\ \varepsilon > 0 \tag{3}$$

$$\dot{r} = \begin{cases} \alpha(\pi - \overline{\pi}) + \beta(Y - \overline{Y}) & \text{if} \quad r > 0 \\ \max[0, \alpha(\pi - \overline{\pi}) + \beta(Y - \overline{Y})] & \text{if} \quad r = 0 \end{cases} \tag{4}$$

$$\dot{\pi}^e = \gamma[\theta(\overline{\pi} - \pi^e) + (1-\theta)(\pi - \pi^e)]\,; \quad \gamma > 0, \quad 0 \le \theta \le 1 \tag{5}$$

where Y = the real national income (real output)> 0, $\overline{Y}$ = the 'natural' output level corresponding to the natural rate of unemployment (fixed)> 0, G = the real government expenditure (fixed)> 0, τ = the marginal tax rate (fixed, $0 < \tau < 1$), M = the nominal money supply > 0, L = the real money demand, p = the price level > 0, $\pi = \dot{p}/p$ = the rate of price inflation, π^e = the expected rate of price inflation, $\overline{\pi}$ = the target rate of price inflation that is set by the central bank, r = the nominal rate of interest ≥ 0, $r - \pi^e$ = the expected real rate of interest.

Equation (1) is the reduced form of the IS equation, which is nothing but the equilibrium condition of the goods market. This model is the 'short run' model in the sense of Keynes(1936) so that the capital accumulation effect of the investment expenditure is not considered. Equation (2) is the LM equation, which is the equilibrium condition of the money market. Equation (3) is a standard type of the 'expectation-augmented price Phillips curve'. Equation (4) is a central bank's interest rate monetary policy rule in the spirit of the 'Taylor rule'(see Taylor,1993). Equation (5) is a formalization of the mixed type inflation expectation formation. We can consider that the parameter θ reflects the 'degree of credibility' of the central bank's inflation targeting. That is, the larger the parameter value θ, the central bank's inflation targeting is considered to be more credible by the public.

Asada, Demetrian and Zimka(2016) reduces this system of equations to the following more compact dynamic system.

$$\dot{r} = \begin{cases} f_1(r, \pi^e; \alpha, \beta, \varepsilon, G, \tau) & \text{if} \quad r > 0 \\ \max[0, f_1(r, \pi^e; \alpha, \beta, \varepsilon, G, \tau)] & \text{if} \quad r = 0 \end{cases} \tag{6a}$$

$$\dot{\pi}^e = f_2(r, \pi^e; \gamma, \theta, \varepsilon, G, \tau) \tag{6b}$$

$$\mu = \frac{\dot{M}}{M} = \varepsilon[Y(r - \pi^e, G, \tau) - \overline{Y}] + \pi^e + \eta_y \frac{Y_{r-\pi^e}\{\dot{r} - \dot{\pi}^e\}}{Y(r - \pi^e, G, \tau)} - \eta_r \frac{\dot{r}}{r} - \eta_\pi \frac{\dot{\pi}^e}{\pi^e} \tag{6c}$$

where $\eta_y > 0$, $\eta_r > 0$, and $\eta_\pi > 0$ are the elasticities of the real money demand with respect to the real national income, the nominal rate of interest and the expected rate of inflation respectively, and we have the following expressions.

$$f_1(r, \pi^e; \alpha, \beta, \varepsilon, G, \tau) = \alpha\{\varepsilon[Y(r - \pi^e, G, \tau) - \overline{Y}] + \pi^e - \overline{\pi}\} + \beta[Y(r - \pi^e, G, \tau) - \overline{Y}] \tag{7}$$

$$f_2(r, \pi^e; \gamma, \theta, \varepsilon, G, \tau) = \gamma\{\theta(\overline{\pi} - \pi^e) + (1 - \theta)\varepsilon[Y(r - \pi^e, G, \tau) - \overline{Y}] \tag{8}$$

It is interesting to note that a set of equations (6a) and (6b) consists of a two-dimensional nonlinear dynamic system that is independent of (6c). Equation (6c) determines the growth rate of the money supply $\mu = \dot{M}/M$ endogenously but it does not influence other part of the system.

It is easy to show that the equilibrium values of this system $(r^*, \pi^{e*}, \pi^*, \mu^*, Y^*)$ becomes as follows.

$$r^* = \rho^*(G, \tau) + \overline{\pi} \tag{9a}$$

$$\pi^{e*} = \pi^* = \mu^* = \overline{\pi} \tag{9b}$$

$$Y^* = \overline{Y} \tag{9c}$$

where $\rho^*(G, \tau)$ is the equilibrium real rate of interest that is determined by

$$Y(\rho^*, G, \tau) = \overline{Y} \tag{10}$$

It follows from the equation (9a) that we have $r* > 0$ if and only if the following inequality is satisfied.

$$\overline{\pi} > -\rho^*(G, \tau) \tag{11}$$

Asada(2010) and Asada, Demetrian and Zimka(2016) assume that the inequality (11) is in fact satisfied. Then, they proved the following proposition.

Proposition 1.

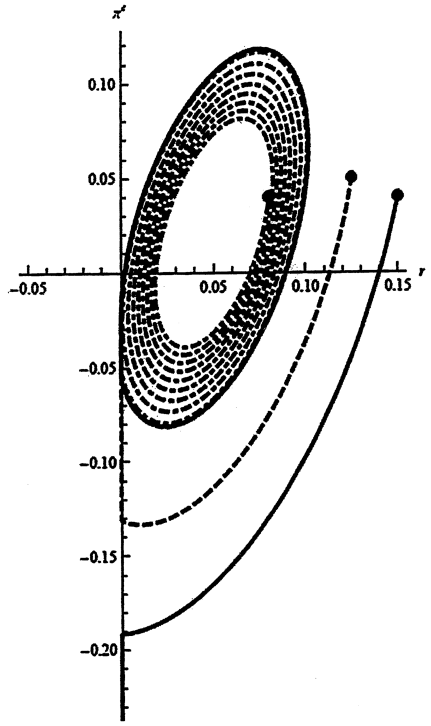
(1) The equilibrium point of the dynamic system (6a) – (6c) is locally stable if the monetary policy parameter values α and β are sufficiently large and the credibility parameter θ is close to 1.

(2) The equilibrium point of the above system is locally unstable if α and β are sufficiently small and θ is close to zero.

(3) At the intermediate range of the parameter values, the cyclical fluctuations occur.[4]

Asada, Demetrian and Zimka(2016) presented some numerical simulations by assuming the monetary policy parameter values $\overline{\pi} = 0.02 = 2\%$ per year, $\alpha = 1/6, \beta = 0.098$,and selecting the credibility parameter θ as a bifurcation parameter. They found that at the critical parameter value $\theta = \theta_0 = 0.539$ the Hopf bifurcation occurs. They also found that the normal equilibrium point is locally unstable for $0 \leq \theta < \theta_0$, and it is locally stable for $\theta_0 < \theta \leq 1$.

Figure 1: Coexistence of a locally stable limit cycle and a path to deflationary depression.



Source: Asada, Demetrian and Zimka (2016) p.158.

Figure 1 illustrates the phase diagram of this system with the nonnegative constraint of nominal rate of interest (r) in case of $\theta = 0.995\theta_0 \simeq 0.536$. In this case, the normal

[4]We can prove this result by using Hopf bifurcation theorem. See Gandolfo(2009), Kuznetsov(2012), and Liu(1994) as for the exposition of Hopf bifurcation theorem. A crucial condition for the occurrence of Hopf bifurcation is that the characteristic equation of the dynamic system at the equilibrium point has a pair of pure imaginary roots at the critical parameter value.

equilibrium point is locally unstable and the time path of the variables $(r(t), \pi^e(t))$ converges to the closed orbit (limit cycle) if the initial condition $(r(0), \pi^e(0))$ is not too far from the normal equilibrium point $(\overline{r}, \overline{\pi})$. On the other hand, the time path of the variables $(r(t), \pi^e(t))$ is led into the 'deflationary depression' with zero nominal rate of interest, decreasing rate of inflation, increasing real rate of interest, and decreasing real national income if the initial condition of the variables is far from the normal equilibrium point. But, it can be shown that the sufficiently active monetary policy (large values of α and β) and / or sufficient credibility of central bank's inflation targeting (the value of θ that is sufficiently close to 1) can contribute to extinguish the deflationary depression and make the normal equilibrium point of this model with natural output level dynamically stable.

3 A Four-dimensional Model of Monetary Stabilization Policy with Private Debt effect

The model that was summarized in the previous section is the simplest version of a series of our dynamic Keynesian models of stabilization policy. Asada, Demetrian and Zimuka(2018) studied more complicated model of monetary stabilization policy that considers private debt effect and capital accumulation effect of investment expenditure.[5] The reduced form of this model is described by the following system of four-dimensional dynamic equations.[6]

$$\dot{d} = \Phi(g) - s_f(r - id) - (g + \pi)d\,; \quad 0 < s_f < 1 \tag{12a}$$

$$\dot{y} = \alpha\{c + \Phi(g) + v - y\}\,; \quad \alpha > 0 \tag{12b}$$

$$\dot{\pi}^e = \gamma[\xi(\overline{\pi} - \pi) + (1 - \xi)(\pi - \pi^e)]\,; \quad \gamma > 0,\ 0 < \xi < 1 \tag{12c}$$

$$\dot{\rho} = \begin{cases} \beta_1(\pi - \overline{\pi}) + \beta_2(y - \overline{y}) & if \quad \rho > 0 \\ \max[0, \beta_1(\pi - \overline{\pi}) + \beta_2(y - \overline{y})] & if \quad \rho = 0 \end{cases}\ ; \beta_1 > 0,\ \beta_2 > 0 \tag{12d}$$

where the following relationships are satisfied.

$$g = I/K = \dot{K}/K = g(r, \rho - \pi^e, d)\,;$$
$$g_r = \frac{\partial g}{\partial r} > 0,\ g_{\rho-\pi^e} = \frac{\partial g}{\partial(\rho - \pi^e)} < 0,\ g_d = \frac{\partial g}{\partial d} < 0 \tag{13}$$

$$r = \beta y\,; \quad \beta = \frac{P}{Y},\ 0 < \beta < 1 \tag{14}$$

$$\pi = \varepsilon(y - \overline{y}) + \pi^e\,; \quad \varepsilon > 0 \tag{15}$$

$$i = i(\rho, d) = \rho + i_1 d\,; \quad i_1 > 0 \tag{16}$$

$$\widetilde{\tau}(y) = \tau_1 y - T_0\,; \quad 0 < \tau_1 < 1,\ T_0 > 0 \tag{17}$$

[5] This model is based on a model that was formulated in Asada(2012, 2014).

[6] In this section, we adopt the symbols that were used in Asada, Demetrian and Zimka(2018). For example, in this section, the symbol r is used to represent the rate of profit rather than the nominal rate of interest.

$$c = (1 - s_1)[(1 - s_f\beta)y - \widetilde{\tau}(y)] + (1 - s_2)i(\rho, d)d + (1 - s_3)\rho b\ ;$$
$$0 < s_1 < 1,\ 0 < s_2 < 1,\ 0 < s_3 < 1,\ b = \text{constant} > 0 \quad (18)$$

The meanings of the symbols are as follows.

D = the stock of firm's nominal private debt, p = the rice level, K = the real capital stock, $d = D/(pK)$ = the private debt-capital ratio,B = the net nominal debt of the consolidated government including the central bank, $b = B/(pK)$ = the government debt-capital ratio (fixed), Y = the real national income (the real output), $y = Y/K$ = the output-capital ratio (the surrogate variable of the capacity utilization rate of the capital stock), $\overline{y}$ = the normal value of output-capital ratio corresponding to the natural rate of employment > 0, C = the real consumption expenditure, $c = C/K$ = the private consumption-capital ratio, G = the real government expenditure, $v = G/K$ = the government expenditure-capital ratio (fixed), T = the real tax, $\widetilde{\tau} = T/K$ = the tax-capital ratio, $\pi = \dot{p}/p$ = the rate of price inflation, $\overline{\pi}$ = the target rate of price inflation that is set by the central bank (fixed) > 0, π^e = the expected rate of price inflation, ρ = the nominal rate of interest of the government debt, i = the nominal rate of interest that is applied to firms' private debt, P = the real profit, $r = P/K$ = the rate of profit, $\beta = P/Y$ = the share of profit in national income (fixed), s_f = firms' internal rate of retention (fixed), (s_1, s_2, s_3) = households' saving rates (fixed), $\Phi(g)$ = the adjustment cost function of investment that has the properties, $\Phi_g = d\Phi/dg \geq 1$, $\Phi_{gg} = d^2\Phi/dg^2 > 0$ (see Uzawa,1969), α = the quantity adjustment speed of the disequilibrium in the goods market, γ = the adjustment speed of the inflation expectation, ξ = the credibility parameter of central bank's inflation targeting, (β_1, β_2) = the monetary policy parameters.

Equation (12a) is the dynamic law of the private debt, which can be derived from the definitional equation $d = D/(pK)$ and the following budget constraint of firms' investment financing.

$$\dot{D} = \Phi(g)pK - s_f(rPK - iD) \quad (19)$$

Equation (12b) is the dynamic multiplier process, which is a Keynesian quantity adjustment process of the disequilibrium in the goods market. Equation (12c) is a mixed type inflation expectation hypothesis that is the same as equation (5) in the previous section. (12d) is a formalization of the interest rate monetary policy rule by the central bank in the spirit of the 'Taylor rule' that is similar to equation (4) in the previous section.

A system of dynamic equations (12a) – (12d) is supplemented by the Keynesian / Minskian investment function with the private debt effect (equation (13)), the definition of the rate of profit (equation (14)), the expectation-augmented price Phillips curve (equation (15)), the relationship between the nominal rates of interest of the private debt and that of the public debt (equation (16)), the tax function (equation (17)), and the Keynesian consumption function (equation (18)).

Asada(2014) and Asada, Demetrian and Zimka(2018) studied the dynamic stability / instability of the normal equilibrium point $E = (d^*, y^*, \pi^{e*}, \rho^*)$ by assuming that the normal equilibrium point such as $d^* > 0$ and $\rho^* > 0$ exists. We can easily see that at the normal equilibrium point we have

$$y^* = \overline{y} > 0, \quad \pi^{e*} = \pi^* = \overline{\pi} > 0 \quad (20)$$

At the normal equilibrium point, we have

$$g^* = g(\beta\overline{y}, r^* - \overline{\pi}, d^*) \quad (21)$$

which is positive under some reasonable conditions. This means that the normal equilibrium in this model can be steady growth equilibrium rather than stationary state. The equilibrium growth rate g^* is determined endogenously in this model. Asada(2014) and Asada, Demetrian and Zimka(2018) proved the following results under some reasonable conditions.

Proposition 2.

(1) The normal equilibrium point of the four-dimensional dynamic system (12a) – (12d) is locally stable if the monetary policy parameters β_1 and β_2 are sufficiently large (central bank's monetary policy is sufficiently active) and the credibility parameter ξ is sufficiently close to 1 (central bank's inflation targeting is sufficiently credible).

(2) The normal equilibrium point of the above system is locally unstable if β_1 and β_2 are sufficiently small (central bank's monetary policy is quite passive) and ξ is close to zero (central bank's inflation targeting is highly incredible).

(3) At the intermediate range of the parameter values, cyclical fluctuations occur through Hopf bifurcation.

In fact, the results of this proposition are qualitatively the same as those of Proposition 1 concerning the simpler two-dimensional model that is summarized in the previous section. Although Asada, Demetrian and Zimka(2018) presented some results of the numerical simulations concerning the four-dimensional dynamic model that is summarized in this section, we omit the explanation of the results of their numerical simulations here.

4 A Six-dimensional Model of Monetary and Fiscal Stabilization Policy Mix with Twin (Public and Private) Debt effects

The four-dimensional model that is summarized in section 3 is much complicated compare with the two-dimensional model that is summarized in section 2. However, even this complicated version is still incomplete in the sense that the public debt-capital ratio $b = B/(pK)$ is assumed to be constant and the government's fiscal stabilization policy is neglected. Asada(2012) and Asada, Demetrian and Zimka(2019) studied more general model of monetary and fiscal stabilization policy mix with twin (private and public) debt accumulation, which consists of the following system of dynamic equations.

$$\dot{d} = \Phi(g) - s_f(r - id) - (g + \pi)d \,; \quad 0 < s_f < 1 \tag{22a}$$

$$\dot{y} = \alpha[c + \Phi(g) + v - y] \,; \quad \alpha > 0 \tag{22b}$$

$$\dot{\pi}^e = \gamma[\xi(\overline{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)] \,; \quad \gamma > 0, \ 0 \le \xi \le 1 \tag{22c}$$

$$\dot{\rho} = \begin{cases} \beta_1(\pi - \overline{\pi}) + \beta_2(y - \overline{y}) & if \quad \rho > 0 \\ \max[0, \beta_1(\pi - \overline{\pi}) + \beta_2(y - \overline{y})] & if \quad \rho = 0 \end{cases} \,; \ \beta_1 > 0, \ \beta_2 > 0 \tag{22d}$$

$$\dot{v} = \sigma[\theta(\overline{y} - y) + (1 - \theta)(\overline{b} - b)] \, ; \quad \sigma > 0, \ \overline{b} > 0, \ 0 \leq \theta \leq 1 \tag{22e}$$

$$\dot{b} = v - \widetilde{\tau}(y) - \frac{\dot{H}}{pK} + (\rho - \pi - g)b \tag{22f}$$

$$\frac{\dot{H}}{pK} = (\pi + g)h + \dot{h} \tag{22g}$$

$$\widetilde{m}(\rho)h = \varphi(\rho)y \, ; \quad \widetilde{m}_\rho = \frac{d\widetilde{m}}{d\rho} > 0, \ \varphi_\rho = \frac{d\varphi}{d\rho} < 0 \tag{22h}$$

These equations are supplemented by the equations (13) – (18) in section 3, where b is a variable rather than constant. The meanings of new symbols are as follows.

H = the nominal high powered money that is issued by the central bank, $h = H/(pK)$ = the high powered money-capital ratio, $\widetilde{m}$ = the money multiplier, $\overline{b}$ = the target value of the public debt-capital ratio that is set by the government, θ = the weight of the employment consideration rather than the public debt consideration in the government's fiscal policy.

Equations (22a) – (22d) are the same as the equations (12a) – (12d) in section 3. Equation (22e) formalizes the government's fiscal stabilization policy rule, which means that the changes of the real government expenditure respond to both the real national income (employment) and the level of public debt. (22f) describes the dynamic of the public debt, which is derived from the following budget constraint of the 'consolidated government' including the central bank.

$$pT + \dot{B} + \dot{H} = pG + \rho B \tag{23}$$

We can derive equation (22g) by differentiating the definitional equation $h = H/(pK)$ by time. (22h) is the equilibrium condition of the money market.[7]

We can express the reduced form of this model by the following six-dimensional system of nonlinear differential equations.

$$\dot{d} = F_1(d, y, \pi^e, \rho) \tag{24a}$$

$$\dot{y} = F_2(d, y, \pi^e, \rho, v, b; \alpha) \tag{24b}$$

$$\dot{\pi}^e = F_3(y, \pi^e; \gamma, \xi) \tag{24c}$$

$$\dot{\rho} = F_4(y, \pi^e; \beta_1, \beta_2) \tag{24d}$$

$$\dot{v} = F_5(y, b; \sigma, \theta) \tag{24e}$$

$$\dot{b} = F_6(d, y, \pi^e, \rho, v, b; \alpha, \beta_1, \beta_2) \tag{24f}$$

Let us write the normal equilibrium point of this system as $E = (d^*, y^*, \pi^{e*}, \rho^*, v^*, b^*)$. We can easily see that at the normal equilibrium point we have

$$y^* = \overline{y} > 0, \quad \pi^{e*} = \pi^* = \overline{\pi} > 0, \quad b^* = \overline{b} > 0, \quad g^* = g(\beta\overline{y}, r^* - \overline{\pi}, d^*) \tag{25}$$

We assume that $g^* > 0$, which means that the normal equilibrium point is the growth equilibrium. Asada(2012) and Asada, Demetrian and Zimka(2019) studied the local stability / instability of the normal equilibrium point and the existence of the cyclical fluctuations around the normal equilibrium point by assuming that a normal equilibrium point such that $d^* > 0$, $\rho^* > 0$, $v^* > 0$ exists. They proved the following results under some reasonable additional conditions.

[7] The left hand side of (22h) is the real money supply per capital stock, and its right hand side is the real money demand per capital stock.

Proposition 3.

(1) The normal equilibrium point of the six-dimensional dynamic system (24a) – (24f) is locally stable if the monetary policy parameters β_1 and β_2 are sufficiently large (central bank's monetary policy is sufficiently active), the credibility parameter ξ is sufficiently close to 1 (central bank's inflation targeting is sufficiently credible), and the fiscal parameter θ that describes the weight of employment consideration is less than 1 but it is sufficiently close to 1 (the weight of public debt consideration $1 - \theta$ is positive but it is sufficiently close to zero).

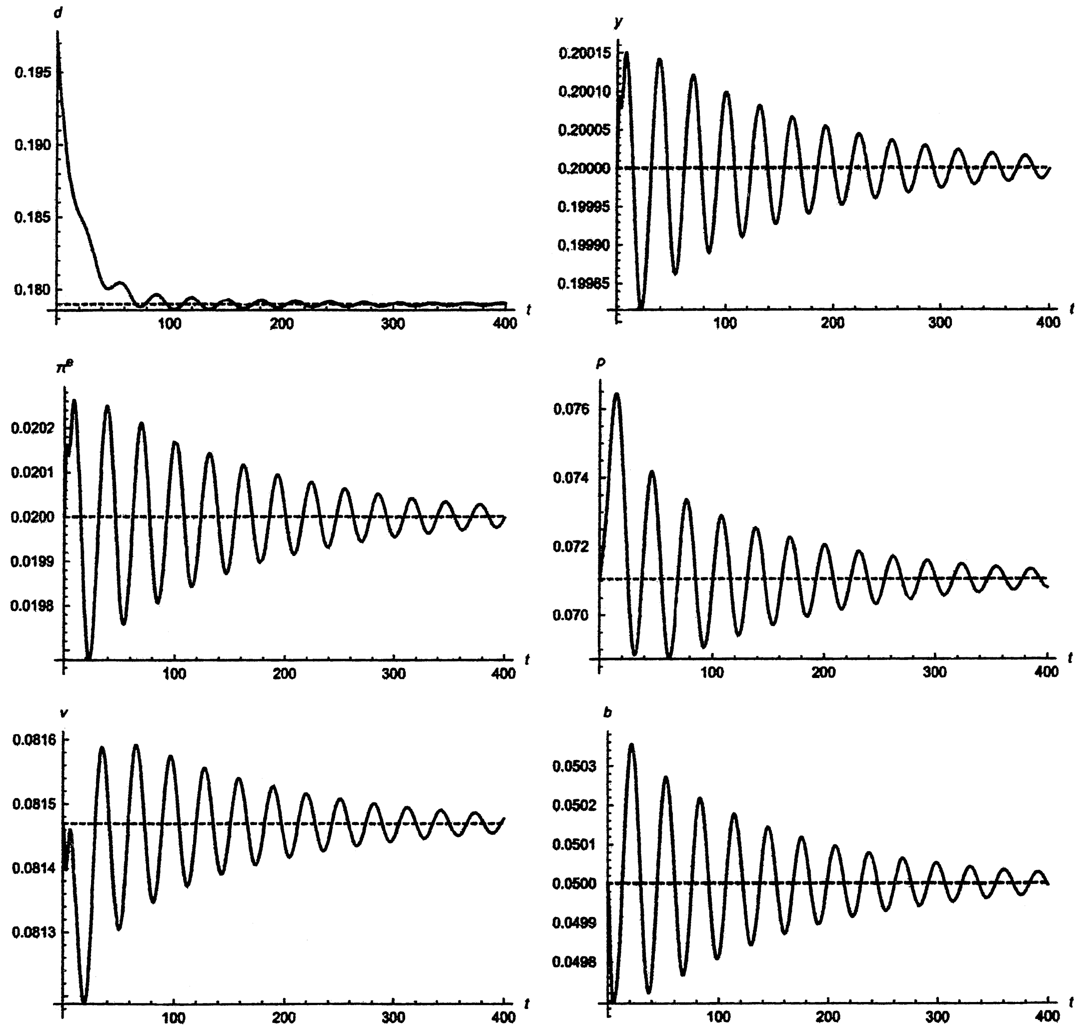
(2) The normal equilibrium point of the above system is locally unstable if β_1 and β_2 are sufficiently small (central bank's monetary policy is quite passive), ξ is close to zero (central bank's inflation targeting is highly incredible), and θ is close to zero (the weight of employment consideration of the fiscal parameter is negligible, which means that its weight of public debt consideration is too high).

(3) At the intermediate range of the parameter values, cyclical fluctuations occur through Hopf bifurcation.

Asada, Demetrian and Zimka(2019) presented some numerical simulations of this six-dimensional dynamic model by assuming the monetary and fiscal parameter values $\overline{\pi} = 0.02$, $\overline{b} = 0.05$, $\beta_1 = 1$, $\beta_2 = 0.2$, $\sigma = 1$, $\theta = 0.7$, and selecting the credibility parameter ξ as a bifurcation parameter. They found that the Hopf bifurcation occurs at $\xi = \xi_0 \simeq 0.484$. They also found that the normal equilibrium point is locally unstable for $0 \leq \xi < \xi_0$, and it is locally stable for $\xi_0 < \xi \leq 1$.

In their numerical example, the normal equilibrium point is locally unstable and the time paths that start near from the normal equilibrium point converge to a closed orbit (limit cycle) in case of $\xi = 0.9\xi_0 \simeq 0.4356$. On the other hand, the normal equilibrium point becomes locally stable in case of $\xi = 1.1\xi_0 \simeq 0.5324$. Figure 2 illustrates the time paths of the main variables in case of $\xi = 1.1\xi_0$. In this case, the time paths that start near from the normal equilibrium point cyclically converge to the normal equilibrium point so that the monetary and fiscal stabilization policy mix is successful.

Figure 2: A case in which monetary and fiscal stabilization policy mix is successful.



Source: Asada, Demetrian and Zimka (2019) p.380.

5 Concluding remarks

In this final section, we shall restate the economic interpretation of the analytical results of the most general six-dimensional model of the monetary and fiscal stabilization policy mix, which was presented in Asada, Demetrian and Zimka(2019). They pointed out that there exist the following intrinsic three destabilizing positive feedback effects $(y \downarrow) \Rightarrow \cdots \Rightarrow (y \downarrow)$ in this model, which are called the 'Fisher debt effect' (FDE), the 'Mundell effect' (ME), and the 'public debt consideration effect' (PDCE) respectively.

$$(y \downarrow) \Rightarrow \pi \downarrow \Rightarrow d = \frac{D}{pK} \uparrow \Rightarrow g \downarrow \Rightarrow (y \downarrow) \quad \text{(FDE)}$$

$$(y \downarrow) \Rightarrow \pi \downarrow \Rightarrow \pi^e \downarrow \Rightarrow (\rho - \pi^e) \uparrow \Rightarrow g \downarrow \Rightarrow (y \downarrow) \quad \text{(ME)}$$

$$(y \downarrow) \Rightarrow \tau \downarrow \Rightarrow b \uparrow \Rightarrow v \downarrow \Rightarrow (y \downarrow) \quad \text{(PDCE)}$$

The mechanism of (FDE) is as follows(see Fisher,1933). The decrease of y induces the decrease of π through price Phillips curve, which induces the increase of the real debt burden of the private firms d, which causes the decrease of y through the decrease of g.

The mechanism of (ME) is as follows. The decrease of π that is caused by the decrease of y induces the decrease of π^e, which causes the increase of the real rate of interest $\rho - \pi^e$, which causes the decrease of y through the decrease of g. This destabilizing effect is strong if the adaptive expectation effect of inflation expectation formation is relatively strong.

The mechanism of (PDCE) is as follows. The decrease of y induces the decrease of the real tax τ, which causes the increase of the public debt burden b. If the 'public debt consideration' of the government expenditure is strong, the increase of b induces the decrease of v, which induces the decrease of y through Keynesian multiplier effect.

On the other hand, there are the following three negative feedback stabilizing effects $(y \downarrow) \Rightarrow \cdots (y \uparrow)$ through monetary and fiscal stabilization policy mix, which are called the 'inflation targeting effect' (ITE), the 'Taylor rule effect' (TRE), and the 'employment consideration effect' (ECE) respectively.

$$(y \downarrow) \Rightarrow \pi \downarrow \Rightarrow \pi^e \downarrow \Rightarrow \pi^e < \overline{\pi} \Rightarrow \pi^e \uparrow \Rightarrow (\rho - \pi^e) \downarrow \Rightarrow g \uparrow \Rightarrow (y \uparrow) \tag{ITE}$$

$$(y \downarrow) \Rightarrow \pi \downarrow \Rightarrow (y < \overline{y}, \pi < \overline{\pi}) \Rightarrow \rho \downarrow \Rightarrow g \uparrow \Rightarrow (y \uparrow) \tag{TRE}$$

$$(y \downarrow) \Rightarrow v \uparrow \Rightarrow (y \uparrow) \tag{ECE}$$

The mechanism of (ITE) is as follows. The decrease of y induces the decrease of π through price Phillips curve, which induces the temporal decrease of π^e. But, the expected rate of inflation π^e begins to increase and it is stabilized toward the target rate of inflation $\overline{\pi}$ if the central bank's inflation targeting is sufficiently credible, which induces the increase of g through the decrease of the real rate of interest $\rho - \pi^e$, and it causes the increase of g. The increase of g causes the increase of y through Keynesian multiplier effect.

The mechanism of (TRE) is as follows. The decrease of y and the resulting decrease of π induce the decrease of nominal rate of interest of the public debt ρ through the Taylor rule monetary policy, which induces the increase of g and the resulting increase of y through Keynesian investment multiplier. However, this stabilizing effect is weak if ρ is already fallen to the level that is near from its lower bound.

The mechanism of (ECE) is as follows. The decrease of y causes the increase of the government expenditure v if the 'employment consideration effect' of the government expenditure is strong, which induces the increase of y through Keynesian multiplier effect.

We omit the economic interpretation of the analytical results of the two-dimensional and the four-dimensional models which were summarized in sections 2 and 3, because these models are the only special cases of the general six-dimensional model.

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