



Entropy-weighted assurance region DEA: an application to healthcare system efficiency assessment

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Abstract

The paper proposes a methodological framework for performance efficiency assessment that integrates entropy weighting with data envelopment analysis (DEA) and assurance region constraints. Entropy-derived lower and upper bounds for input and output weight ratios are constructed, providing a more objective, data-driven way to limit weight flexibility without relying on additional information or expert judgment. The proposed model addresses key limitations of classical DEA - particularly its tendency to assign extreme or zero weights that can artificially overestimate the efficiency of low-performing units. By incorporating a scalable parameter, the model enables a controlled narrowing of the admissible weight space. The methodology is applied to the assessment of healthcare systems in European countries. Four aggregated input variables cover the staff providing healthcare, the scope and availability of healthcare services and the population behaviour. The survival to 65 years of age and the expected healthy life years at age 65 as significant health outcomes covering the whole human life are considered outputs. Efficient healthcare systems with satisfactory outcomes are identified, and the healthcare systems are ranked based on adjusted efficiency scores. The results reveal that, under stricter assurance region constraints, the DEA frontier becomes more robust, and efficiency scores better reflect acceptable performance. The approach provides a flexible tool for analysts and policymakers seeking to evaluate efficiency with greater realism and policy alignment.

Keywords Data envelopment analysis · Assurance region constraints · Entropy weights · Healthcare system performance · European countries

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1 Introduction

This work contributes to the ongoing development of performance evaluation methods by offering a model for efficiency assessment that integrates entropy weighting with data envelopment analysis (DEA) and assurance region (AR) constraints. It bridges the gap between unconstrained, overly permissive models and rigid, expert-defined systems by offering a data-driven, adaptable, and scalable methodology that combines the advantages of DEA, entropy weighting, and AR constraints.

The efficiency of a decision-making unit (DMU) is measured as its relative productivity within a group of comparable units that transform inputs of the same kind to outputs of the same nature. The efficiency score of a DMU is calculated as a ratio of a weighted sum of its outputs to a weighted sum of its inputs, provided that the efficiency scores of all units in the group for any set of weights do not exceed the value of one.

DEA is designed to derive the optimal weights of the input and output variables for the individual DMUs in the group from the observed data on their activities so that each unit is assigned the maximum efficiency score. Since the efficiency score is calculated based on a linear optimisation program, a DMU may reach its efficiency score with very small or even zero values of some weights. However, such weights are inappropriate when all inputs and outputs are significant, with a limited level of mutual substitution.

To eliminate the effect mentioned above, weight restrictions are used to design the AR DEA model. Weight restrictions prevent zero weights as well as too large differences among the weights assigned to the same input or output variable, which may extremely enlarge the impact of a strong point of a DMU as well as reduce the impact of its weak point on its efficiency score.

The setting of weight restrictions is a key step in AR DEA models. The restrictions may reflect some prior information (e.g. the prices of inputs and/or outputs), expert opinions, or the information provided by the observed input and output data (see Cooper et al. 2011).

In our paper, we contribute to the third approach by suggesting a method for setting weight restrictions based on the information entropy of the data set. The benefit of using entropy lies in its objectivity, since the assessment of the weight for a certain variable determines the relative importance of the variable without direct involvement of the decision maker.

The entropy weights of the variables are in inverse proportion to their entropy values. According to information theory, if the data of some variable vary considerably, it provides much useful information, and its entropy value is low. It follows that the choice of variables affects their individual entropy weights.

Our motivation for using entropy-based weights in DEA comes from the study of efficiency of national healthcare systems. They use a wide variety of health resources to provide healthcare and thus support the improvement of health outcomes of the population. A DEA model for the assessment of healthcare system efficiency natu-

rally involves health resources as inputs and health outcomes as outputs. All input and output variables are significant and specific, and none of them can be fully substituted for others. Therefore, none of the variables can be neglected in the efficiency assessment by assigning a zero weight. However, a straightforward application of weight restrictions faces considerable difficulties. It is difficult to determine the prices of health resources appropriate for international comparison and even impossible to estimate the price of health outcomes. Expert views on the importance of inputs and outputs may also be rather different. This reasoning supports deriving weight restrictions endogenously from the observed input and output data.

Moreover, by traditional DEA models, a DMU can be considered efficient also in the case that it exhibits low levels of outputs along with low levels of inputs. But healthcare systems with this performance can hardly be accepted as efficient by stakeholders, and the analysis does not provide any recommendations for their improvement. Since healthcare delivery is designed to improve the health of the population, it is necessary to emphasise the priority of outputs in the applied DEA model. The entropy-based weight restrictions allow us to prioritise health outcomes by choosing output variables with lower entropy values compared to the input variables. This choice leads to higher entropy weights for output variables in the AR DEA model.

The aim of this study is to develop a methodological framework that integrates entropy-based weighting with the AR DEA to enhance the objectivity and robustness of efficiency measurement. Moreover, by changing a controllable parameter incorporated in the weight restrictions, we can justify the range of assurance region to identify such a group of efficient DMUs that may be acceptable by stakeholders.

The paper is organised as follows. The literature on weight restrictions and previous applications of entropy in DEA is summarised in Sect. 2. The research methodology is introduced in Sect. 3. Based on the methodology proposed, the performance of national healthcare systems of European countries is evaluated in Sect. 4. Conclusions and policy implications are presented at the end of the paper.

2 Previous research on weight restrictions and the use of entropy in DEA

In this section, we provide an overview of previous studies covering weight restrictions and applications of entropy in DEA.

2.1 Weight restrictions in DEA

To avoid zero values for some weights, various weight restrictions may be incorporated into the multiplier formulation of the DEA model (see, e.g., Cooper et al. 2007). Weight restrictions can be applied for individual inputs and/or outputs, as well as for pairs of input/output variables. An overview of literature on the use of weight restrictions in DEA covering the theoretical aspects of the approaches used and the resulting models, their properties, interpretations, and possible problems and weaknesses can be found in Allen et al. (1997) and Thanassoulis et al. (2004).

In Allen et al. (1997), direct weight restrictions are categorised into three types: assurance regions type I (ARI, first proposed by Thomson et al. 1986), assurance regions type II (ARII, introduced by Thomson et al. 1990) and absolute weight restrictions (first proposed by Dyson and Thanassoulis 1988).

The use of weight restrictions may face some obstacles. Suppose that we have a set of n DMUs each of which utilizes m inputs to produce s outputs.

Imposing absolute weight restrictions

$$L_i \leq v_i \leq U_i \text{ and/or } L_r \leq u_r \leq U_r, \quad (i = 1, 2, \dots, m, \text{ and } r = 1, 2, \dots, s), \quad (1)$$

for input weights v_i and for output weights u_r directly on DEA models may lead to infeasibility, or the efficiency scores may be underestimated (see, for example, Allen et al. 1997; Podinovski and Athanassopoulos 1998).

The problems mentioned above do not occur in DEA models that use ARI (see Charnes et al. 1990; Thompson et al. 1990; Podinovski 2001) with weight restrictions

$$l_{1,i} \leq \frac{v_i}{v_1} \leq u_{1,i} \text{ and/or } L_{1,r} \leq \frac{u_r}{u_1} \leq U_{1,r}, \quad (i = 2, \dots, m, \text{ and } r = 2, \dots, s), \quad (2)$$

where one of the input weights (say v_1) and/or one of the output weights (say u_1) were chosen as numeraires.

In ARII DEA models, restrictions linking the weights of input and output variables are also incorporated, i.e.

$$\mathcal{L}_{i,r} \leq \frac{u_r}{v_i} \leq \mathcal{U}_{i,r}, \quad (i = 1, 2, \dots, m, \text{ and } r = 1, 2, \dots, s). \quad (3)$$

Thompson et al. (1990) and Thompson and Thrall (1994) pointed out a limitation associated with the use of ARII in DEA models, which is the possibility that the model is infeasible for some or all DMUs. This observation may justify the small number of empirical applications that used DEA models with ARII (see, for example, Thanassoulis et al. 1995, who applied ARII for English Perinatal Care units).

The limitation of ARII can be overcome by nonlinear DEA models (see Thompson and Thrall 1994; Khalili et al. 2010). Tracy and Chen (2005) also addressed this issue for a generalised form of weight restrictions, which covers all forms of weight restrictions described in the literature.

Setting the bounds in the assurance region constraints is one of the key issues in these approaches. The methodological aspects of the specification and the use of weight restrictions are nicely summarised in Cooper et al. (2011).

2.2 Application of entropy in DEA

In our paper, restrictions linking input and output weights are established to emphasise the relative importance of inputs and outputs based on the entropy of the variables.

The concept of entropy, derived from statistical physics and thermodynamics, was introduced into information theory by Shannon (1948) to describe the disorder

and uncertainty of a system by the theory of probability and statistics. Information entropy evaluates the degree of disorder of a system. The greater the degree of dispersion or differentiation of a given data set, the lower the entropy, and more information can be derived from the data set. Thus, the data set with a lower entropy is more valuable from the point of view of total information on the units considered. Based on the entropy of different data sets in the system, the entropy weight of each data set can be calculated to assess its relative contribution to the total information provided to the system (Zeleny 1982).

Several authors used entropy-based weights to compare the information values of different data sets of the same system, for example, in the assessment of water quality indices (Gorgij et al. 2017; Sun et al. 2019; Zhu et al. 2020) or in the detection of significant features of power lines (Tan et al. 2021).

Some literature has combined the DEA method with entropy. In the paper by Soleimani-Damaneh and Zarepisheh (2009), entropy was used to aggregate efficiency scores from different DEA models. Yang et al. (2010) proposed a statistical approach to detect influential observations in DEA in which entropy was used to detect the change in the distribution after some DMU is removed from the assessed group. Several researchers applied the entropy approach to increase the discriminatory power of DEA models and create comprehensive rankings (see, for example, Bian and Yang 2010; Jayaraman and Srinivasan 2014; Xie et al. 2014; Lo Storto 2016; Nissi and Sarra 2018).

Qi and Guo (2014) proposed a methodology using entropy to aggregate different sets of optimal weights into a common set of weights. Then they evaluated the DMUs with the common set of weights. Davoudabadi et al. (2020) proposed an integrated framework combining entropy, DEA, and principal component analysis to derive weights and rankings in complex multi-criteria environments. Deng et al. (2021) introduced entropy-based variable selection in DEA, showing that entropy can be used to reduce redundancy and retain variables with higher discriminatory power.

Wu et al. (2011, 2012) introduced a method for aggregating cross-efficiency with entropy. Si and Ma (2019) suggested a DEA cross-efficiency ranking method based on the evaluation of relative entropy and grey relational analysis. The approach combines the characteristics of the two methods to determine their relative closeness. Han et al. (2018) developed the environmental DEA cross model based on the information entropy used in the cross-evaluation matrix to construct the entropy distance between the self-evaluation index and the cross-evaluation index. Pan et al. (2024) presented the RACE (Regret cross-efficiency evaluation) method, integrating attitudinal entropy to reflect uncertainty and preference variation in peer evaluations. Similarly, Chen et al. (2024) incorporated a distance entropy function into DEA cross-efficiency models under prospect theory, capturing variability and risk attitudes. Sun et al. (2025) developed an entropy-weighted cross-efficiency DEA model for environmental and socio-economic vulnerability assessments, where entropy objectively determines indicator weights.

Extensions of other DEA models have also drawn on entropy concepts. Xu (2022) combined Delphi-derived expert weights with anti-entropy measures in a super-efficiency DEA model, thereby enhancing robustness in efficiency evaluation. Yang et al. (2024) proposed a meta two-stage parallel dynamic DEA model where entropy

weighting supports the allocation of resources and captures dynamic interactions across stages.

Applications of entropy in the DEA can also be found in Shuai and Wu (2011), Hsiao et al. (2011), and Zhou et al. (2013). Mete and Belgin (2022) used the entropy weights of the input and output variables to set lower bounds of weight restrictions for the ARI CCR model with two inputs and two outputs. They used entropy weights to set the lower bounds of weight restrictions and thus to emphasise the weight of one input and one output variable against the input and output numeraire, respectively. This may lead to a notable imbalance in the importance of the variables in the model, and efficiency scores are significantly influenced by the selection of numeraires. However, to our knowledge, no studies have yet used entropy to set the bounds for weight restrictions in ARII DEA models.

3 The research methodology

We propose a methodology using entropy to find appropriate ARII weight restrictions for a DEA model.

The calculation process for setting the weight restrictions for the ARII DEA model can be formulated in the following four steps. The first three steps are in accordance with the well-known entropy weight method (Zeleny 1982). Our contribution lies in Step 4, where we specify both the lower and upper bounds of the weight restrictions of ARII DEA model. This setting allows us to control the differences among weights and thus balance their importance in the model.

Step 1: Data normalization. We suppose that the values of input and output variables are positive and that there are no outliers in the data set that may artificially increase the variability. For convenience of comparison, the input data x_{ij} ($i = 1, 2, \dots, m$) and the output data y_{rj} ($r = 1, 2, \dots, s$) of DMU _{j} ($j = 1, 2, \dots, n$) are normalized. The normalized value of the i -th input and the r -th output of the j -th DMU is denoted as p_{ij}^{IN} and p_{rj}^{OUT} , respectively, and they are calculated as follows:

$$\begin{cases} p_{ij}^{IN} = \frac{x_{ij}}{\sum_{j=1}^n x_{ij}} \\ p_{rj}^{OUT} = \frac{y_{rj}}{\sum_{j=1}^n y_{rj}} \end{cases} \quad (4)$$

Step 2: Entropy value calculation. The entropy is calculated separately for each input variable and for each output variable:

$$\begin{cases} E_i^{IN} = -\frac{1}{\ln n} \sum_{j=1}^n p_{ij}^{IN} \cdot \ln p_{ij}^{IN} \\ E_r^{OUT} = -\frac{1}{\ln n} \sum_{j=1}^n p_{rj}^{OUT} \cdot \ln p_{rj}^{OUT} \end{cases} \quad (5)$$

The range of entropy values is $[0, 1]$. The lower the entropy, the more importance and a higher weight is assigned to the variable.

Step 3: Entropy weight calculation. The weight of the input or output variable is defined in the following way:

$$\left\{ \begin{array}{l} w_i^{IN} = \frac{1-E_i^{IN}}{\sum_{a=1}^m (1-E_a^{IN}) + \sum_{b=1}^s (1-E_b^{OUT})} = \frac{1-E_i^{IN}}{m+s-\sum_{a=1}^m E_a^{IN}-\sum_{b=1}^s E_b^{OUT}} \\ w_r^{OUT} = \frac{1-E_r^{OUT}}{\sum_{a=1}^m (1-E_a^{IN}) + \sum_{b=1}^s (1-E_b^{OUT})} = \frac{1-E_r^{OUT}}{m+s-\sum_{a=1}^m E_a^{IN}-\sum_{b=1}^s E_b^{OUT}} \end{array} \right. \quad (6)$$

Step 4: Assurance region bounds setting. Based on the entropy weight ratios, the upper and lower bounds of the assurance region are determined by (7)–(9). The size of the assurance region depends on the parameter α that ranges from 0 to 1.

$$\frac{w_i^{IN}}{w_1^{IN}} (1-\alpha) \leq \frac{v_i}{v_1} \leq \frac{w_i^{IN}}{w_1^{IN}} (1+\alpha), \quad \text{where} \quad \frac{w_i^{IN}}{w_1^{IN}} = \frac{1-E_i^{IN}}{1-E_1^{IN}}, \quad (i=2, \dots, m) \quad (7)$$

$$\frac{w_r^{OUT}}{w_1^{OUT}} (1-\alpha) \leq \frac{u_r}{u_1} \leq \frac{w_r^{OUT}}{w_1^{OUT}} (1+\alpha), \quad \text{where} \quad \frac{w_r^{OUT}}{w_1^{OUT}} = \frac{1-E_r^{OUT}}{1-E_1^{OUT}}, \quad (r=2, \dots, s) \quad (8)$$

$$\frac{w_1^{OUT}}{w_1^{IN}} (1-\alpha) \leq \frac{u_1}{v_1} \leq \frac{w_1^{OUT}}{w_1^{IN}} (1+\alpha), \quad \text{where} \quad \frac{w_1^{OUT}}{w_1^{IN}} = \frac{1-E_1^{OUT}}{1-E_1^{IN}} \quad (9)$$

In accordance with (2), conditions (7) and (8) specify the ARI weight restrictions that link each input weight and each output weight to the corresponding numeraire. Condition (9) is a special case of (3) that specifies the ARII weight restriction for the two numeraires. Due to the nature of the entropy approach, conditions (7)–(9) link all input and output weights to each other and thus are sufficient to ensure a non-zero value of each weight.

Note that the parameter α plays a crucial role in setting the power of the weight restrictions. The lower the parameter value, the smaller the size of the assurance region and the tighter the restrictions, resulting in a smaller number of efficient DMUs and lower efficiency scores of the inefficient ones identified by the DEA model. The impact of the choice of value α on the model results needs to be analysed to fit the model for the given input and output data set. The sensitivity analysis allows us to select the appropriate value of α for which the group of efficient peers meets specific requirements on the assessment.

4 Empirical study: assessment of national healthcare systems

In this section, we apply the developed methodology to assess the performance of national healthcare systems of European countries. For performance assessment, we can consider the national healthcare system as a DMU that is under the control of national health policy.

The input and output variables selected for the efficiency assessment are introduced in Sect. 4.1. We also perform a segmentation of countries by output data in Sect. 4.2. to avoid the identification of healthcare systems with unsatisfactory health outcomes as efficient. In Sect. 4.3, we discuss the selection of a proper DEA model and present the results of the analysis.

A national healthcare system is built to support the health conditions of the population, which are traditionally measured by health outcomes such as life expectancy

and mortality. Personal and capital resources of healthcare systems are used to ensure availability and provision of health services and thus improve health outcomes. From this perspective, it is natural to study the efficiency of national healthcare systems, that is, to ask whether health resources and provided services are efficiently used to achieve desirable health outcomes of the population.

Several studies have been published that apply DEA to compare national healthcare systems among themselves. In most of them, the labour and capital resources used in healthcare systems, along with the health expenditure per capita, are considered inputs, while the health outcomes of the population are outputs (see Afonso and St Aubyn 2005). Bhat (2005) considered practising physicians, nurses, inpatient beds, and pharmaceuticals as inputs to treat populations of various age groups and examine the efficiency of healthcare delivery systems in OECD countries. Hadad et al. (2013) compared healthcare systems using two models, using discretionary inputs (physicians' density, inpatient bed density, and health expenditure per capita) in one model and inputs beyond the control of healthcare systems (health expenditure per capita, GDP per capita and consumption of fruits and vegetables per capita) in another model.

Hsu (2013) used DEA to evaluate the efficiency of spending health expenditures by selected countries in Europe and Central Asia to support health outcomes (life expectancy at birth, infant mortality rate) and provide healthcare (immunisation against measles). Grausová et al. (2014) applied DEA to analyse the development of healthcare systems in the Visegrád group compared to other European countries. Dlouhý (2015) used a two-stage DEA approach to evaluate healthcare system efficiency in two steps: the efficiency of health resources (number of physicians in full-time equivalents and total number of hospital beds) to produce health services (the number of hospital discharges and the number of outpatient contacts) in Stage 1, and the efficiency of health services to support the total population and its health measured by the life expectancy at birth in years in Stage 2.

Afonso and St. Aubyn (2011) applied a two-stage DEA-Tobit analysis to OECD countries, introducing socioeconomic variables in the second stage to explain efficiency differences. Ahmed et al. (2019) applied a similar two-step framework to Asian health systems, while Top et al. (2020) used DEA followed by Tobit regression to explore efficiency determinants across African countries.

More recent works have extended the methodology. Gavurova et al. (2021) applied a dynamic network DEA to OECD healthcare systems and Pereira et al. (2022) proposed a network DEA for assessing pandemic responses. Both studies illustrate how DEA can be adapted to reflect multi-stage processes or temporal dynamics. Dlouhý (2023) revisited the issue of non-homogeneity in cross-country efficiency evaluations, highlighting the importance of combining different analytical methods and methodological refinements to ensure comparability. In this spirit, Dlouhý and Havlík (2024) combined DEA with MCDA to evaluate 28 European systems, demonstrating the benefits of hybrid models for policy interpretation. Manavgat and Audibert (2024) further advanced the two-stage DEA-Tobit framework by applying bias-corrected DEA with double-bootstrap truncated regression to OECD countries before and during the COVID-19 pandemic, identifying health policy and socioeconomic factors as key determinants of resilience and efficiency.

However, it is generally accepted that the impact of a healthcare system on the health outcomes of inhabitants is limited and that the differences in health outcomes between countries cannot be fully explained without considering a wider context, i.e. the lifestyle of the population or the state of the environment. The context includes a wide variety of factors, and it is difficult to accurately assess the contribution of individual factors to improving or worsening health outcomes in individual countries. In addition, all these factors may not be closely related to the provision of healthcare. Therefore, it is meaningful to consider a simple (one-stage) DEA model that incorporates a relevant combination of health resources, health services, and population behaviour characteristics as inputs and health outcomes as outputs. For this purpose, it may be useful to apply composite indicators designed by experts in the health sector that include several important factors to provide a more complex picture of the national context.

4.1 An input-output set to assess healthcare systems efficiency

We study the efficiency of national healthcare systems within a group of 29 European countries including EU members (except for Cyprus), Norway, Switzerland, and the United Kingdom. These countries are in a relatively small geographic area, and all of them deploy similar and closely interconnected socioeconomic systems. Additionally, reliable and comparable data on their healthcare provision and health outcomes are available, including composite health indices.

The inputs in our analysis cover the human resources involved in providing healthcare and the scope and availability of healthcare services. We focus on the provision of healthcare and the determinants of health closely related to cardiovascular diseases. As reported by *Health at a Glance* (OECD/European Union 2020), circulatory diseases continue to be the leading cause of death in the EU, accounting for approximately 37% of all deaths in 2017, and mortality rates from circulatory diseases are approximately 40% higher among men than among women. We take this observation into account when selecting DEA output variables that reflect the health outcomes of the male population.

The level of healthcare services is expressed by three aggregated input variables that are derived from the Euro Health Consumer Index (EHCI, Bjornberg 2017) and the Euro Heart Index (EHI, Garrofé et al. 2016). Both EHCI and EHI include several dozens of indicators grouped into a few subdisciplines. A country is assigned a value of 1, 2 or 3 for each indicator, depending on the level of compliance with the given criteria. A higher value represents better performance that is expected to have a positive impact on health outcomes. The value of our aggregated input for each country is calculated as the mean of values of all indicators in the corresponding subdiscipline of EHCI or EHI, excluding non-available and non-applicable indicators for some countries.

The input variable Prevention is calculated based on the corresponding subdiscipline of EHCI, which includes indicators capturing important preventive measures applied within the healthcare systems, but also indicators measuring some aspects of the behaviour or lifestyle of the population with a significant positive or negative impact on the health outcomes. Of course, the values of the behaviour and life-style

indicators are not directly controlled by the healthcare system. However, they may be affected by appropriate government policies and human decision makers in a reasonably short time.

The input variables Procedures and Access are calculated based on the subdisciplines of EHI (Procedures and Access to care, respectively), reflecting the fact that there has been a general advancement of therapies and procedures, contributing to reducing heart mortality in Europe. The achievement of good outcomes for people suffering from any cardiovascular disease is dependent on the provision of well-organised and coordinated services that draw on the knowledge and skills of healthcare professionals working in primary and secondary care. Additionally, people with cardiovascular disease or at high cardiovascular risk need early detection and treatment with counseling and medications, as appropriate.

Since the structure of healthcare professionals varies across countries depending on their roles and responsibilities, human resources are represented by a unique input variable Staff, calculated as the total number of physicians, nurses, and midwives per 1,000 inhabitants, based on data provided by the World Bank (2020).

Regarding the outputs, we do not rely on traditional variables such as the infant mortality rate and life expectancy at birth. Instead, we take the survival rate to age 65 (which extends the infant mortality rate to cover a longer period of human life) and healthy life years (which better reflects the quality of population health as a main target of providing healthcare). To increase the discrimination power of the analysis, we modify the output variables. In accordance with Grausová and Hužvár (2020), we use the number of survived males per one non-survived male up to age 65. In addition, the number of healthy life years of males at age 65 is applied in the further analysis. The threshold of 65 years in both outputs coincides with the current retirement age, which is 64.5 years in the European Union in 2022 for a male who entered the labour force at age 22 (OECD 2023).

The input and output variables used in our analysis are briefly summarised in Table 1, and their data are taken from 2016. Note that higher values of the inputs mean better healthcare provision and a better environment for health support, and higher values of the outputs mean better health conditions for the population. All variables are independent of the size of a country.

Table 1 Variables used in efficiency assessment

	Variable	Meaning
Inputs	Prevention ^a	Captures a combination of preventive measures applied within the healthcare systems, and the lifestyle of the population, based on the Euro Health Consumer Index, subdiscipline Prevention
	Procedures ^b	Estimates healthcare performance contributing to reducing heart mortality, based on the Euro Heart Index, subdiscipline Procedures
	Access ^b	Evaluates the access to treatment/care for acute and non-acute patients with cardiovascular disease, based on the Euro Heart Index, subdiscipline Access to care
	Staff ^c	Total number of physicians, nurses, and midwives per 1000 inhabitants
Outputs	Survival to 65 ^c	Survival rate to 65 of males (number of survived per one non-survived)
	HLY at 65 ^d	Healthy life years of males at age 65

Source: ^aBjornberg (2017), ^bGarroř et al. (2016), ^cWorld Bank (2020) and ^dEurostat (2025)

The outputs are naturally expected to be influenced by the quality of the health-care, as well as the behaviour of the population measured by selected inputs.

4.2 Segmentation of countries by output data

To better understand the differences and similarities among countries, we will group them by their health outcomes. We use the median values of the output variables to divide all countries into four segments (see Fig. 1). The countries with both outputs greater than or equal to the median are located in Segment I.

The lowest values of Survival to 65 are seen in Lithuania and Latvia (below 2 survived per 1 non-survived), the highest values in Switzerland, Sweden, Italy, and Malta (all over 8 survived per 1 non-survived). The lowest values of Healthy life years at 65 appear in Latvia and the Slovak Republic (less than 5 years) and the highest in Norway and Sweden (over 15 years).

Note that countries with both outputs less than the median that lie in Segment IV may be hardly acceptable as efficient although many of them (Bulgaria, Croatia, Czech Republic, Estonia, Latvia, Lithuania, Romania, and Slovak Republic) exhibit relatively low inputs.

4.3 Ranking of national healthcare systems

In this section, we apply ARII models introduced in Sect. 3 to calculate the efficiency scores and rank the assessed countries for the combination of input and output variables specified in Sect. 4.1.

The choice of underlying DEA model is based on the following assumptions.

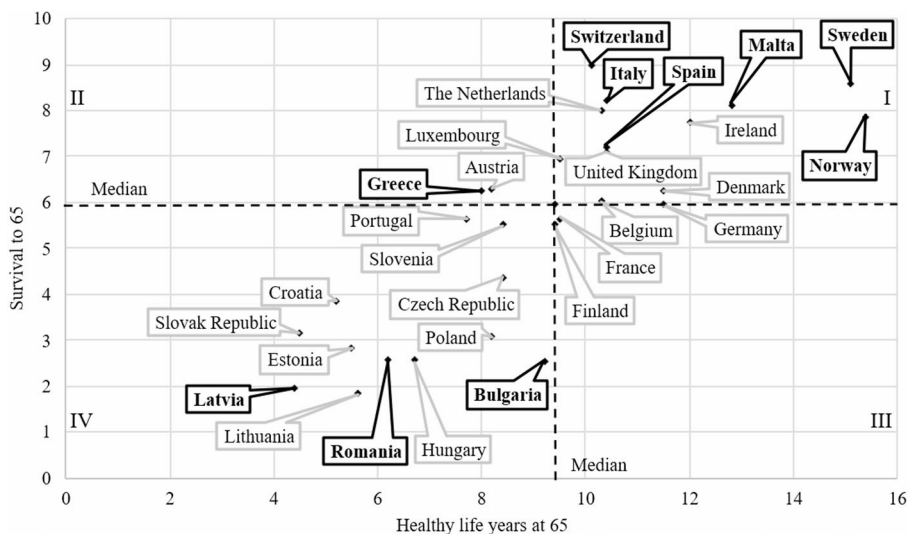


Fig. 1 Segmentation of European countries by health outcomes. The highlighted countries are efficient by the SBM-O-V DEA model; see Table 4

- The primary objective of the healthcare system is to improve health outcomes, i.e., the outputs of the model. Therefore, the calculated efficiency score used for ranking should measure the potential of a country for output improvement.
- The position of some countries within the assessed group is rather different for the two outputs (see Fig. 1). Thus, non-proportional improvements determined by slacks are naturally expected from the DEA model.
- The character of the inputs and outputs as well as the mutual comparisons between countries advocate the use of variable returns to scale (VRS), as countries with better health outcomes can hardly reach an additional increase in the outcomes that would be proportional to the increase in their inputs.

As a result of the previous considerations, we apply the output-oriented slacks-based DEA model (Tone 2001) with variable returns to scale (SBM-O-V). DEA-Solver-Pro Version 15.b software (Cooper et al. 2007) is used for computations related to the DEA technique.

The SBM-O-V model identifies ten efficient countries that are highlighted in Fig. 1. However, not all of them really represent best practices. Bulgaria, Latvia, and Romania belong to Segment IV, that is, among countries with unsatisfactory health outcomes. Their efficiency indicated by SBM-O-V results from the natural character of DEA, since they have very low levels of inputs, too. Therefore, the analysis does not provide any recommendations for improving their healthcare systems, although their performance is clearly insufficient. Furthermore, Bulgaria and Romania are peers for numerous inefficient countries, including some with much better health outcomes. Greece is a special case, since it is good in one output but poor in the other. Latvia, Norway, and Switzerland are peers only for themselves, so they do not affect the efficiency scores of inefficient countries.

As we have already argued, the assessment of efficiency of healthcare systems requires a more careful approach. We apply the assurance region methodology to avoid assigning zero weights for weak inputs and outputs of individual healthcare systems, to reduce weight unbalances across inputs and outputs, and to strengthen the importance of outputs. We follow the procedure for setting the weight restrictions described in Sect. 3. The optimisation programs of proposed models are summarised in the Appendix.

Table 2 shows the entropy values and entropy weights of the input and output variables calculated using the proposed methodology. The weight of the variable Survival to 65 with the lowest entropy of the data is as high as 0.362. The input variables based on the EHI and EHCI indices have the highest level of entropy; therefore, they receive the smallest weights, which are about half of the weight of the input variable Staff. Higher weights of the output variables are convenient since they emphasise the priority we set to the outputs.

Table 2 Entropy values and entropy weights of variables used for DEA

Variable	Prevention	Procedures	Access	Staff	Survival to 65	HLY at 65
Entropy value	0.995	0.993	0.994	0.987	0.975	0.986
Entropy weight	0.074	0.095	0.086	0.182	0.362	0.201

Based on the entropy weights, we construct the assurance regions (ARII) for the selected slack-based DEA model (SBM-AR-O-V). ARII bounds are defined by formulas (7)–(9), and we use the control parameter α varying from 90% to 10% (with a gradual reduction of 10% points). The corresponding DEA models are denoted by E90 to E10. Table 3 lists the countries identified as efficient by the DEA models and indicates how often each appears in the reference sets of inefficient countries. ARII models with stronger weight restrictions naturally admit a smaller number of efficient countries.

The efficiency scores of the individual countries for each DEA model are given in Table 4. The countries are grouped by segments and alphabetically sorted within them.

Since the entropy-based weights for the outputs are higher compared to the entropy weights of the inputs, the ARII DEA models naturally favour countries with higher outputs. Note that a consecutive strengthening of weight restrictions (from E90 to E10) leads to a decrease in efficiency scores while the variability of the scores measured by standard deviation gradually increases (see Table 4).

A natural question arises as to which of the proposed models is the most suitable for the assessment and ranking of the performance of healthcare systems. If good health outcomes are the priority of the stakeholder, we prefer models with efficient countries in Segment I or II.

The sensitivity analysis in Tables 3 and 4 provides an opportunity to choose a suitable ARII model based on the preferences of the stakeholder. A model can be considered suitable if it identifies an adequate number of efficient countries but at the same time excludes the countries with unsatisfactory health outcomes from the efficient set. On the other hand, if the number of efficient countries is too small, the efficiency scores of inefficient countries are strongly influenced by the small group of efficient ones, and thus they may not adequately reflect the national and regional specificities.

The E90 model identifies seven efficient countries, but still includes two Segment IV countries (Bulgaria, Romania). On the other hand, only two countries are efficient by model E10. If we restrict the efficient set to Segment I countries, model E30 with three efficient countries (Italy, Malta, and Sweden) is recommended.

Table 3 Frequency of the efficient countries in the reference set of inefficient ones

	SBM-O-V	E90	E80	E70	E60	E50	E40	E30	E20	E10
Bulgaria	5	6	–	–	–	–	–	–	–	–
Greece	4	7	8	8	8	6	1	–	–	–
Italy	5	9	14	14	14	14	14	15	14	15
Latvia	0	–	–	–	–	–	–	–	–	–
Malta	15	16	13	13	13	12	12	12	12	17
Norway	0	–	–	–	–	–	–	–	–	–
Romania	10	2	–	–	–	–	–	–	–	–
Spain	1	–	–	–	–	–	–	–	–	–
Sweden	12	10	11	11	11	11	11	11	11	–
Switzerland	0	0	–	–	–	–	–	–	–	–

Countries inefficient by the model are assigned –

Table 4 Efficiency scores by proposed DEA models

Segment	Score	SBM-O-V	E90	E80	E70	E60	E50	E40	E30	E20	E10
I	Belgium	0.759	0.739	0.738	0.738	0.738	0.737	0.737	0.736	0.736	0.734
	Denmark	0.816	0.795	0.792	0.790	0.789	0.789	0.788	0.788	0.788	0.783
	Germany	0.737	0.727	0.726	0.725	0.724	0.723	0.721	0.720	0.718	0.705
	Ireland	0.910	0.861	0.853	0.848	0.846	0.844	0.843	0.843	0.842	0.836
	Italy	1	1	1	1	1	1	1	1	1	1
	Luxembourg	0.742	0.741	0.740	0.739	0.739	0.738	0.737	0.735	0.728	0.719
	Malta	1	1	1	1	1	1	1	1	1	1
	The Netherlands	0.839	0.837	0.837	0.836	0.835	0.834	0.833	0.831	0.827	0.822
	Norway	1	0.963	0.952	0.940	0.927	0.913	0.898	0.881	0.862	0.839
	Spain	1	0.981	0.978	0.974	0.970	0.961	0.954	0.948	0.942	0.934
	Sweden	1	1	1	1	1	1	1	1	1	0.998
	Switzerland	1	1	0.865	0.812	0.798	0.790	0.781	0.769	0.753	0.732
	United Kingdom	0.907	0.889	0.888	0.888	0.888	0.887	0.886	0.886	0.883	0.877
II	Austria	0.690	0.689	0.689	0.688	0.688	0.687	0.687	0.681	0.673	0.664
	Greece	1	1	1	1	1	1	1	0.902	0.858	0.833
III	Finland	0.707	0.656	0.642	0.634	0.631	0.629	0.626	0.623	0.620	0.612
	France	0.717	0.714	0.713	0.712	0.711	0.710	0.709	0.708	0.706	0.704
IV	Bulgaria	1	1	0.854	0.678	0.614	0.582	0.560	0.533	0.512	0.497
	Croatia	0.631	0.566	0.529	0.504	0.492	0.486	0.481	0.475	0.463	0.450
	Czech Republic	0.768	0.665	0.629	0.614	0.607	0.603	0.601	0.599	0.598	0.597
	Estonia	0.642	0.554	0.493	0.464	0.451	0.443	0.438	0.434	0.431	0.428
	Hungary	0.469	0.459	0.448	0.439	0.434	0.432	0.430	0.427	0.418	0.410
	Latvia	1	0.606	0.481	0.446	0.431	0.422	0.406	0.381	0.364	0.342
	Lithuania	0.491	0.376	0.339	0.319	0.312	0.310	0.307	0.298	0.289	0.280
	Poland	0.652	0.642	0.641	0.640	0.639	0.638	0.628	0.597	0.570	0.550
	Portugal	0.746	0.739	0.738	0.737	0.737	0.736	0.730	0.725	0.713	0.700
	Romania	1	1	0.755	0.643	0.598	0.575	0.551	0.517	0.489	0.463
	Slovak Republic	0.528	0.482	0.449	0.422	0.410	0.403	0.399	0.396	0.388	0.375
All	Slovenia	0.776	0.687	0.674	0.670	0.669	0.669	0.668	0.667	0.667	0.658
	Average	0.811	0.771	0.739	0.721	0.713	0.708	0.703	0.693	0.684	0.674
	St. Dev.	0.168	0.185	0.185	0.190	0.194	0.196	0.198	0.198	0.200	0.202
	Min	0.469	0.376	0.339	0.319	0.312	0.310	0.307	0.298	0.289	0.280

In model E80, the countries with low outputs Bulgaria and Romania are excluded from the efficient set, and their efficiency scores sharply drop. Hence, only countries from Segment I (Italy, Malta, and Sweden) and Segment II (Greece) remain in the reference set of inefficient ones. As a result, the efficiency scores of inefficient countries for which Bulgaria and Romania were efficient peers also decrease. From this point of view, the model E80 can be applied.

However, when moving from E80 to E30, the scores of most segment IV countries (with exception of Slovenia, Portugal, Hungary, and Czech Republic) still decrease sharply (visualised in Fig. 2), resulting in changing their positions in the ranking (see Fig. 3). It means that their scores are overestimated by traditional DEA model and

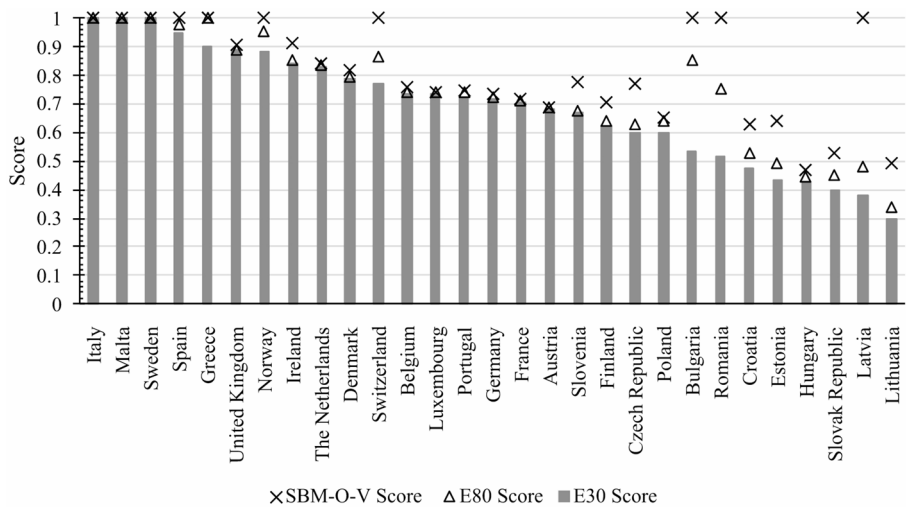


Fig. 2 Comparison of efficiency scores of national healthcare systems calculated by the proposed DEA models

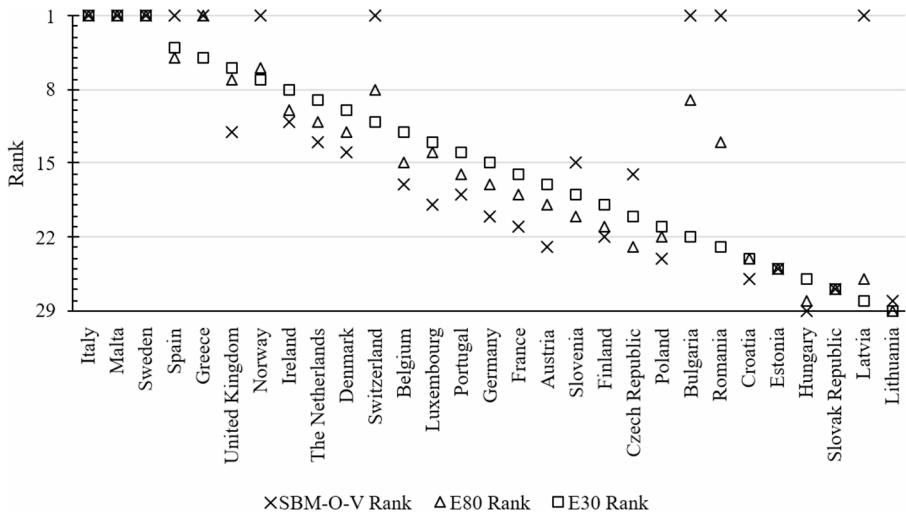


Fig. 3 Ranking of national healthcare systems by the proposed DEA models

applied AR approach makes them more realistic. On the contrary, the application of stronger AR constraints has only a slight impact on the scores of countries in Segments I–III, with the exceptions of Norway, Switzerland and Greece that turn to be inefficient when stronger AR constraints are used.

5 Conclusions and policy implications

This study introduces a novel methodological framework that enhances traditional DEA by integrating entropy-weighted constraints within an AR II model. The primary contribution lies in the construction of entropy-derived lower and upper bounds for input and output weights, allowing for a more objective and data-driven way to limit weight flexibility without relying on subjective expert decision. The methodology is also applicable in both types of assurance regions if exogenous information on the relative importance of input or output variables is ambiguous or missing.

The proposed model addresses key limitations of classical DEA, particularly its tendency to assign extreme or zero weights that can artificially inflate the efficiency of low-performing units. By incorporating entropy-based bounds and a scalable parameter, the model enables controlled narrowing of the admissible weight space. This provides a flexible, yet principled approach for balancing discrimination power and fairness in efficiency evaluations.

Applied to the assessment of healthcare systems in 29 European countries, the model demonstrates its practical value. The results reveal that, under stricter entropy-weighted constraints, the DEA frontier becomes more robust, and efficiency scores better reflect acceptable output performance. The ability to adjust the model's sensitivity through the control parameter further enhances its policy relevance, as it allows analysts to explore how changes in weight flexibility affect the composition of efficient units.

However, several methodological and empirical limitations should be noted. First, entropy values are purely data-driven and reflect the degree of variability in the dataset that may not align with theoretical or policy relevance of variables. Second, the results are inherently sensitive to the choice of the input–output set and to the presence of outliers in the data set. Third, the assurance region parameter α introduces an additional degree of freedom, but no universal criterion exists for selecting its “optimal” value, and different parameter choices can produce substantially different rankings. Finally, as with all ARII formulations, the incorporation of multiple weight-linking constraints increases the risk of infeasibility in more complex datasets and may limit the model's scalability. Moreover, the entropy-based approach may reduce transparency for policymakers compared to models based on expert judgment or price information. These limitations indicate that the model should be applied in conjunction with contextual knowledge.

Nevertheless, the methodology provides a transparent, replicable framework for evaluating performance efficiency – particularly in environments where full weight flexibility may produce misleading results. The entropy constraints ensure that inefficient systems cannot be deemed efficient simply due to low input usage, promoting more realistic and actionable performance assessments.

The control parameter enables a tailored policy analysis, allowing stakeholders to explore various evaluation scenarios, facilitating informed policy debates about trade-offs between resource use and health outcomes. Although applied here to healthcare systems, the methodology is generalisable and may be extended to other public sector evaluations where fairness, transparency, and quality of the outputs are critical.

Appendix

DEA Models Used in Sect. 4.

Envelopment form of the output-oriented SBM-O-V model.

$$\begin{aligned}
 \text{(SBM-O-V) Minimize } \rho_o &= \frac{1}{1 + 1/s \sum_{r=1}^s s_r^+ / y_{ro}} \\
 \text{subject to } \sum_{j=1}^n \lambda_j x_{ij} &\leq x_{io}, & i = 1, \dots, m, \\
 \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ &= y_{ro}, & r = 1, \dots, s, \\
 \sum_{j=1}^n \lambda_j &= 1, & j = 1, \dots, n, \\
 \lambda_j &\geq 0, & j = 1, \dots, n, \\
 s_r^+ &\geq 0, & r = 1, \dots, s.
 \end{aligned} \tag{A1}$$

(SBM-O-V) can be transformed into the program below after incorporating the positive scalar variable t .

$$\begin{aligned}
 \text{(SBM-O-V t) Minimize } t & \\
 \text{subject to } t + 1/s \sum_{r=1}^s t s_r^+ / y_{ro} &= 1 \\
 \sum_{j=1}^n \lambda_j x_{ij} + s_i^- &= x_{io}, & i = 1, \dots, m, \\
 \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ &= y_{ro}, & r = 1, \dots, s, \\
 \sum_{j=1}^n \lambda_j &= 1, & j = 1, \dots, n, \\
 \lambda_j &\geq 0, & j = 1, \dots, n, \\
 s_i^- &\geq 0, & i = 1, \dots, m, \\
 s_r^+ &\geq 0, & r = 1, \dots, s, \\
 t &\geq 0.
 \end{aligned} \tag{A2}$$

Defining new variables $S_i^- = t s_i^-$, $S_r^+ = t s_r^+$ and $\Lambda_j = t \lambda_j$ the (SBM-O-V t) model becomes the following linear program.

$$\begin{aligned}
& \text{(SBM-O-V, LP)} \quad \text{Minimize } t \\
& \text{subject to} \quad t + 1/s \sum_{r=1}^s S_r^+ / y_{ro} = 1 \\
& \quad \sum_{j=1}^n \Lambda_j x_{ij} + S_i^- = tx_{io}, \quad i = 1, \dots, m, \\
& \quad \sum_{j=1}^n \Lambda_j y_{rj} - S_r^+ = ty_{ro}, \quad r = 1, \dots, s, \\
& \quad \sum_{j=1}^n \Lambda_j = t, \quad j = 1, \dots, n, \\
& \quad \Lambda_j \geq 0, \quad j = 1, \dots, n, \\
& \quad S_i^- \geq 0, \quad i = 1, \dots, m, \\
& \quad S_r^+ \geq 0, \quad r = 1, \dots, s, \\
& \quad t \geq 0.
\end{aligned} \tag{A3}$$

Multiplier form of the output-oriented SBM-O-V model

$$\begin{aligned}
& \text{(SBM-O-V, DP)} \quad \text{Maximize } \xi \\
& \text{subject to} \quad \xi - \sum_{r=1}^s y_{ro} u_r + \sum_{i=1}^m x_{io} v_i + v_0 = 1, \\
& \quad \sum_{r=1}^s y_{rj} u_r - \sum_{i=1}^m x_{ij} v_i - v_0 \leq 0, \quad j = 1, \dots, n, \\
& \quad v_i \geq 0, \quad i = 1, \dots, m, \\
& \quad u_r \geq \frac{\xi}{s} \left[\frac{1}{y_{ro}} \right], \quad r = 1, \dots, s, \\
& \quad v_0 \text{ free.}
\end{aligned} \tag{A4}$$

Adding the following weight restrictions, we obtain the output-oriented assurance region type II SBM-AR-O-V model used in our analysis.

The weight restrictions of the inputs can be added by incorporating the constraints

$$\begin{aligned}
& v_1 l_{1,i} - v_i \leq 0, \\
& -v_1 u_{1,i} + v_i \leq 0, \\
& i = 2, \dots, m.
\end{aligned} \tag{A5}$$

Weight restrictions of outputs can be added by incorporating the constraints

$$\begin{aligned}
& u_1 L_{1,r} - u_r \leq 0, \\
& -u_1 U_{1,r} + u_r \leq 0, \\
& r = 2, \dots, s.
\end{aligned} \tag{A6}$$

Weight restrictions that link input and output numeraires can be added by incorporating the constraints

$$\begin{aligned}
& v_1 \mathcal{L}_{11} - u_1 \leq 0, \\
& -v_1 \mathcal{U}_{11} + u_1 \leq 0.
\end{aligned} \tag{A7}$$

Envelopment form of the output-oriented assurance region type II SBM-AR-O-V model used in our analysis

$$\begin{aligned}
 & \text{(SBM-AR-O-V, LP) Minimize } t \\
 & \text{subject to} \\
 & t + 1/s \sum_{r=1}^s S_r^+ / y_{ro} = 1, \\
 & \sum_{j=1}^n \Lambda_j x_{1j} + S_1^- = tx_{1o} - \sum_{i=2}^m \pi_{l,i} l_{1,i} + \sum_{i=2}^m \pi_{u,i} u_{1,i} + \sigma_{\mathcal{L}} \mathcal{L} - \sigma_{\mathcal{U}} \mathcal{U}, \\
 & \sum_{j=1}^n \Lambda_j x_{ij} + S_i^- = tx_{io} - \pi_{l,i} + \pi_{u,i}, \quad i = 2, \dots, m, \\
 & \sum_{j=1}^n \Lambda_j y_{1j} - S_1^+ = ty_{1o} + \sum_{r=2}^s \tau_{L,r} L_{1,r} - \sum_{r=2}^s \tau_{U,r} U_{1,r} - \sigma_{\mathcal{L}} + \sigma_{\mathcal{U}}, \\
 & \sum_{j=1}^n \Lambda_j y_{rj} - S_r^+ = ty_{ro} + \tau_{L,r} - \tau_{U,r}, \quad r = 2, \dots, s, \\
 & \sum_{j=1}^n \Lambda_j = t, \quad j = 1, \dots, n, \\
 & \Lambda_j \geq 0, \quad j = 1, \dots, n, \\
 & S_i^- \geq 0, \quad i = 1, \dots, m, \\
 & S_r^+ \geq 0, \quad r = 1, \dots, s, \\
 & t \geq 0, \\
 & \pi_{l,i} \geq 0, \pi_{u,i} \geq 0, \quad i = 2, \dots, m, \\
 & \tau_{L,r} \geq 0, \tau_{U,r} \geq 0, \quad r = 2, \dots, s, \\
 & \sigma_{\mathcal{L}} \geq 0, \sigma_{\mathcal{U}} \geq 0,
 \end{aligned} \tag{A8}$$

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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